ABOUT «TIME» IN MECHANICS OF DEFORMABLE BODY

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Endochronic (with own, internal «time») approach is considered for the description of processes of viskoelasticity and destruction in the mechanics of deformable bodies.

The concept of «time» is used in mechanics of continuous media as «usual» physical (laboratory) «time» and as some functional «endochronic» [1] (own, internal (rendered, transformed, modified, thermodynamic etc.)) «time» convenient for the description of nontrivial, nonlinear processes of deformation (of creep and relaxation) [2-6], and also destruction [5] of bodies under various physical-chemical-mechanic influence (temperature, moist, ionizing exposure, aggressive medium, static and dynamic monotonous and nonmonotonous loading etc). Various «simple» and «complicated» «analogies» (correspondences, hypothesis) are applied. In mechanics of dynamic destruction «structural» («incubation») [7] time is used, introduction of which bases on the idea of V.V. Novojilov and other contributors of the existence of a structural (scale) level of destruction dependent on a kind of medium and loading, and, therefore, - some characteristic time of destruction at the given level.

In the present work the uniform invariant «endochronic» approach is presented for the account of various effects on media properties ander deformation and destruction based on transformed time, scales of which can be generally complicated nonlinear functionals. The main mathematical model of such description bases on nonlinear summation of sections of the transformed time and linear superposition of mechanical aftereffects, with damping «memory», in its space. In this space indicial equations of viskoelasticity look like hereditary quasilinear integral equations of Bolzman-Volterra, that is convenient in many applications, in particular, for the circulation of the equations of creep and relaxation, for the use of the equality of transformed «times» for stress and deformation. Equations of this tipe allow to apply as a base the linear theory of accumulation of damages and power criterion for the evaluation of destruction and strength. The given approach represents a possibility to evaluate visibly and visually complicated nonlinear processes of viskoelastic deformation and destruction under various effects. We can evaluate properties of media and structural levels by functions of viskoelasticity, basic curves of strength and scales of transformed time.

In **isothermal case of equations of creep and relaxations** under monoaxial expansion (compression, pure shift), - mechanical effect, look like unified [8] linear integrated ratio byVolterra in the scale of transformed (own) «time» [6]:

$$e(t)=\overline{P}\sigma,\ \sigma(t)=\overline{R}e,\ P=Q_1,\ R=Q_2,\ \sigma=m_1,\ e=m_2,$$

$$\overline{Q_i}m_i=\int_{0^-}^{\xi^{m_i}}Q_i\left[\xi^{m_i}-\varsigma\right]\dot{m}_i(\varsigma)d\varsigma=\int_{0^-}^{t}Q_i\left[\xi^{m_i}(t,t)-\xi^{m_i}(t,\theta)\right]\dot{m}_i(\theta)d\theta, \quad (1)$$

where e - deformation; σ - stress; t - laboratory time; $\overline{P}$, P, $\overline{R}$ and R -, accordingly, operators and functions of creep and relaxation; ξ^{m_i} - transformed «times», the expressions for which can be various. Let's consider some of them:

$$\xi^{m_i}(t,t)-\xi^{m_i}(t,\theta)=\overline{G}^{m}(t,\theta)=\int_{\theta}^{t}G^{m_i}\left[t-\tau,m_i(\tau),\bar{f}^{m_i}(\dot{m}_i)\right]d\tau,$$

$$\bar{f}^{m_i}(\dot{m}_i)=\int_{\tau}^{t}f^{m_i}\left[t-\rho,m_i(\rho)\right]\dot{m}_i(\rho)d\rho,\quad f^{m}(0)=0. \quad (2)$$

Here $\overline{G}^{m}$, G^{m}, $\bar{f}^{m_i}$, f^{m_i} - operators and scales "of «time». The dependence of G^{m} on three arguments allows to use convenient in applience «hierarchy» of different rank scales g^{m_i}, G^{m_i} and G^{m}, difined one through the other, used for appropriate kinds of media and loading. In particular, it is possible to present G^{m} in the form

$$G^{m_i}\left[t-\tau,m_i(\tau),\bar{f}^{m_i}(\dot{m}_i)\right]=G^{m_i}\left[t-\tau,m_i(\tau)\right]\left\{1-F^{m_i}\left[\bar{f}^{m_i}(\dot{m}_i)\right]\right\},$$

$$G^{m_i}\left[t-\tau,m_i(\tau)\right]=g^{m_i}\left[t-\tau,m_i(\tau)\right]+\frac{\partial g^{m_i}\left[t-\tau,m_i(\tau)\right]}{\partial(t-\tau)}(t-\tau), \quad (3)$$

where F^{m_i} - function of a material equal to zero when $m_i\le m_i^*$ - in the field of linear viskoelasticity, and when $1-F^{m_i}\left[\ldots\right]\le 0$, in case of nonlinear deformation.

$F^{m_i}\left[0\right]=0$. The analysis of experimental and estimated data of polymeric materials has shown, that it is possible to apply to them functions of f^{m_i} and F^{m_i} kind

$$f^{m_i}\left[t-\tau, m_i(\tau)\right]=a_k^{m_i}\,,\; F^{m_i}\left[\bar{f}^{m_i}(\dot{m}_i)\right]=\left[f^{m_i}(\dot{m}_i)\right]^{b_k^{m_i}}\,,\; k=1,2\,. \quad (4)$$

Coefficients $a_k^{m_i}$ and $b_k^{m_i}$ - constants of a media dependent on the sign $\dot{m}_i$: $k=1$ when $\dot{m}_i \geq 0$; $k=2$; in case $\dot{m}_i < 0$. The application of factors $a_k^{m_i}$ and $b_k^{m_i}$ allows to take into account the difference of viskoelasticity properties of polymers in nonlinear area under the change of the sign of the derivative $\dot{m}_i$, in particular under expansion and compression. The use of scales g^{m_i}, G^{m_i}, and Γ^{η} enables us to describe viskoelasticity behaviour with accelerated (consolidation), «normal» and slow (disconsolidation) responses, to evaluate properties of media. Conducted experiments on various classes of isotropic and anisotropic polymeric materials (thermoreactive, thermoplastic, amorphous linear, netting, partially crystalline) allow to consider scales physical features of a medium «feeling» its structure and response to effect. The correlation of scales with other functions appropriate to the structure of materials is established. It is necessary to note some other, including «unusual», properties of transformed «time». In linear area it coincides with the laboratory time and during the process it increases, and in nonlinear area it differs from the laboratory time, it can increase and decrease (it is connected with nonlinear transformation, either increase or decrease of loading or deformation), - but it is never less than the laboratory time. In general transformed «time» is <u>relative</u>, <u>inhomogeneous</u> and <u>non-isotropic</u> [6].

It follows from (1) - (3), that when $m_i(\rho)=H(\rho)m_i^0$, $m_i^0=const$ ($H(\rho)$ - the single function Hevisaid) is executed: $\bar{f}^{m_i}=0$, $\Gamma^{\eta}=G^{\eta}$,

$$\xi^{m_i}-\varsigma=g^{m_i}(t-\theta, m_i^o)(t-\theta)\,,\; \xi^{m_i}(t)=g^{m_i}(t, m_i^o)t\,,$$

$$e(t)=P\left[g^{\sigma}(t,\sigma^0)t\right]\sigma^0\,,\; \sigma(t)=R\left[g^{e}(t,e^0)t\right]e^0\,.$$

Using these degenerated formulas it is possible to define functions P, g^{σ}, R and g^{e}, and through ratioes of the general type, varying parameters and comparing results of calculations and experiments, - the remaining functions and coefficients.

In **non isothermal processes** $\varepsilon(t) = e(t) + \varepsilon^{T}(t)$, (5)

where $\varepsilon(t)$ - full, and $\varepsilon^{T}(t)$ - thermal deformation. The mechanical deformation of thermocreep can be defined througt (1) - (4), using the temperature-temporary correspondence (we shall limit ourselves by the «simple» correspondence executed for the most part of materials):

$$e\left[t(\xi^{T,\sigma})\right] = P(0)\sigma(\xi^{T,\sigma}) + \int_{0^+}^{\xi^{T,\sigma}} \dot{P}\left[\xi^{T,\sigma} - \varsigma\right]\sigma(\varsigma)d\varsigma, \qquad (6)$$

$$\xi^{T,\sigma} - \varsigma = \overline{G}^{T,\sigma}(t,\theta) = \int_{\theta}^{t} G^{T,\sigma}\left[t-\tau, \sigma(\tau), \bar{f}^{\sigma}(\dot{\sigma}), T(\tau)\right]d\tau =$$

$$= \int_{\theta}^{t} g^{T}\left[T(\tau)\right] G^{\sigma}\left[\int_{\tau}^{t} g^{T}\left[T(\tau)\right]d\rho, \sigma(\tau), \bar{f}^{\sigma}(\dot{\sigma})\right]d\tau. \qquad (7)$$

Here $\xi^{T,\sigma}$ - transformed on temperature T and stress σ «time»; $g^{T}(T)$ - scale of the time, transformed on temperature. The equation of thetmorelaxation (6), (7) looks similarly. The results of experiments have shown, that the scale g^{T} is sensitive to the structure of a medium, in particular, it «reacts» to physical transitions (glass and high elasticity condition).

The mutual **circulation of indicial equations** (1) - (4) is based on a quasilinear unified kind of ratio (1) and logical physical supposition of the equality of transformed (unified) «times» of creep and relaxation. Whence follow equations:

$$\xi^{\sigma}(t) = \xi^{e}(t) = \xi(t), \qquad (8)$$

$$\overline{P}R(\xi) = P(0)R(\xi) + \int_{0^+}^{\xi} \dot{P}(\xi - \varsigma)R(\varsigma)d\varsigma = 1, \qquad (9)$$

(or $\overline{R}P = 1$), applying which, it is possible trough functions of creep to define functions of relaxation, and vice versa. Certainly, this circulation is approximate, as the severe mathematical mutual convertibility of ratioes (1) - (4) is not proved. It is confirmed by experimental data.

As it was stated, the **criteria of destruction (strength)** under endochronic approach can be received, by the replacement in ratioes of the linear theory of the laboratory time by transformed time. In case of monoaxial expansion the normalized criteria, in the damaging form [9], can have a various look:

$$\omega(t) = \overline{\omega}^{n_k} t \leq 1, \qquad (10)$$

Where ω - damaging; $\overline{\omega}^{n_k}$ - integrated, as in the hereditary theory of damaging or work of deformation, operator of damaging; $n_1 = t$, $n_2 = \sigma$, $n_3 = e$, $n_4 = \sigma e$. The operators of damaging are obtained by application of the appropriate theories of strength and endochronic operators $\overline{Q}_i$ (1) - (4), connecting σ, e.

For example, in correspondence with the **theory of accumulation of damages** (integral of Baily [2, 9]),

$$\omega(t) = \overline{\omega}^t t = \overline{\mathrm{T}}^{\xi^\sigma} t = \int_0^{\xi^\sigma} \mathrm{T}_r\left[\sigma(\varsigma)\right] d\varsigma = \int_0^{\xi^\sigma} \frac{d\varsigma}{\xi_r\left[\sigma(\varsigma)\right]} =$$

$$= \int_0^t \frac{G^\sigma\left[t - \rho, \sigma(\rho), \bar{f}^\sigma(\sigma)\right]}{\xi_r\left[\sigma(\rho)\right]} d\rho, \qquad (11)$$

and according to the **theory of a power type**

$$\omega(t) = \overline{\omega}^{\sigma e} t = \overline{\mathrm{U}}^{\xi^\sigma}(\sigma de) = \int_0^{\xi^\sigma} \mathrm{U}_r^\sigma(\xi^\sigma - \varsigma)\sigma(\varsigma)\frac{de(\varsigma)}{d\varsigma} d\varsigma =$$

$$= \int_0^{\xi^\sigma} \frac{\sigma(\varsigma)}{U_r^\sigma(\xi^\sigma - \varsigma)} \frac{de(\varsigma)}{d\varsigma} d\varsigma = \int_0^t \frac{\sigma(\rho)}{U_r^\sigma(\xi^\sigma - \varsigma)} \frac{de(\rho)}{d\rho} d\rho =$$

$$= \overline{\omega}^\sigma t = \int_0^{\xi^\sigma} \frac{\sigma(\varsigma)}{U_r^\sigma(\xi^\sigma - \varsigma)} \frac{d(\overline{P}\sigma)}{d\varsigma} d\varsigma. \qquad (12)$$

Here $\overline{H}^{\xi^{\sigma}}$, $H_r(\sigma)$ - operator and its function; $\xi_r(\sigma_r)$ - function (curve) of durability (long strength) when $\sigma = const$; $\overline{U}^{\xi^{\sigma}}$, $U_r^{\sigma}(\varsigma)$ - operator and its function; $U_r^{\sigma}(\xi_r)$ - function of durability power when $\sigma = const$. $U_r^{\sigma}(\xi_r)$ - poorly chaning function of support, or constant in general (absolute invariant). $\xi_r(\sigma_r)$ - heavily changing function. Therefore, in a number of cases it is more convenient to apply criteria of a power type, though criteria like accumulation of damages are easier outwardly for calculations. As is noted, the criteria of destruction (damage) (10) - (12) can be represented in a different form.

The structural - temporary criterion (STC) of dynamic destruction of an impulse type looks like [7]:

$$\frac{1}{\tau}\int_{t-\tau}^{t}\sigma(\rho)d\rho \le \sigma_c, \qquad (13)$$

where τ - structural (incubation) time; t - time of destruction; σ - stress; σ_c static strength (temporary resistance). For the comparison with other theories we shall reduce the hurdle rate of a criterion (13) to zero, by the replacement of the variable of the integration: $\rho = a + b\varphi$ (when $\rho = t - \tau$, $\varphi = 0$, and when $\rho = t$, $\varphi = t$). Besides we shall present it (13) as «damage» (10):

$$\omega(t) = \overline{\omega}^{\varsigma}_{\tau,\sigma_c} t = \frac{1}{t\cdot\sigma_c}\int_{0}^{t}\sigma(\varsigma)d\varphi \le 1. \qquad (14)$$

It is possible to say, that it is endochronic, with own, internal time $\varsigma = a + b\varphi$ ($a = t - \tau$, $b = \tau / t$), form of STC. A function of support of STC, is $\sigma_r\left\{\frac{t}{\tau}\left[H(t) - H(t-\tau)\right] + H(t-\tau)\right\} = \sigma_c$. Comparing (14) to (11) and (12) it is easy to define, that some correlation can be established between them. STC is a corollary not only nonlinear (endochronic), but also linear variants of a criterion of accumulation of damages and power (under appropriate choice of their functions of support). It is noted in [7], that - the structural time τ characterizes the scale (structural) level of dynamic destruction. It is necessary to note, that (successfully called) «scales» of transformed «times» characterize structural (scale) levels of the processes of destruction accumulation and destruction.

The approaches, considered in the given work, and criteria have a significant universality stipulated by endochronic «complicated» integrated «hereditary» description, and allow to evaluate viscoelastics effects (creep and relaxation), strength and predisposition to destruction of materials with various properties, under various kinds of monotonous and nonmonotonous, static and dynamic loading, proceeding from different, but interconnected, physically reasonable hypothesises of viscoelasticity and strength, under the uniform, invariant methodology, - different from the approaches of many contributors, who seem tobe exlectic in the account of various factors of processes. Introduction of a special kind of inner «times», having nontrivial properties, takes the most essential part in descriptions like that.

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