

ON THE UNIVERSAL QUANTUM-GRAVITATION EQUATION AND DIRAC HYPOTHESIS

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ABSTRACT

The authors propose a gravitational electro-magnetic vortex (GEM-vortex) as a basic model for the Universe having a vacuum field fluctuation structure, in which, h , c , and G are three unvarying universal fundamental constants. The Einstein's mass-energy relation is to hold with a new form of quantum gravitation. The angular frequency of electro-magnetic vortices give rise to a concentration of energy, *i.e.*, it forms the gravitational center of mass, while the gravitational potential tends to smear the concentrated energy.

By this model, the gravitation field and electro-magnetic field may be combined into a single orthogonal structure of GEM field. The difference between Dirac's and Einstein's theories of gravitational constant G may then be reconciled. The antithesis of vacuum space and matter may fade away, and the whole of physics would become a completely enclosed intellectual system. Furthermore, the Primordial Planck particle, Primordial Planck-Cosmos, Expanding Annex Universe, Present Cosmos, Chaos Universe and their classical universe equations, universal quantum-gravitation equations, the conservation constant of Universe mass-energy, the constant of Universal Quantum-Gravitation and all kinds of interaction coupling constants may all come out consistent with observations and experiments.

It is a challenging problem for physicists of our time to answer how one can succeed in line with A. Einstein's suggestion^{<1>} to unite the gravitational field and the electro-magnetic field into a single structure, and to rely on which to solve the Dirac's "Large Number Hypothesis"^{<2>} puzzle of gravitation and cosmology^{<3>} which are presently unexplained.

Planck Cosmos, Universal Quantum-Gravitation Equation, and Classical Universe-Equation

It is the light's wave-particle dualistic nature that determines vacuum's gravitational electro-magnetic (G•EM) field structure. It is just as the left- and right-hand rules of electric motor and generator, showing that the E•M and G fields are perpendicular to each other and thus form an orthogonal basis structure.

By analogy to the viewpoint of the vortex theory^{<4>}, proposed in 1867-1887 by William Thomson (Lord Kelvin), and also of similar ideas considered by J.A. Wheeler^{<5>}, who has noted that gravitational and electromagnetic field equations admit solution that describes a reasonably stable concentration of energy and that such a gravitational electromagnetic entity (he called the "geon") bears an analogy to the vortex ring of classical hydrodynamics. We propose that the primary particle of the Universe may be considered as a basic energy packet of the gravitational electromagnetic (G•EM) vortex ring in the Universe vacuum. Then, we made three fundamental assumptions as follows:

(A) The basic packet model of the whirl energy in the gravitational electromagnetic field (G•EM-field) for the Universe vacuum consists of the dualistic gravitational electro-magnetic (G•EM) vortex, having a vacuum field fluctuation structure. In contrast to the customary view of the dualistic gravitational electromagnetic wave (G•EM-wave) of the photon which is the propagation form of the energy in the gravitational electromagnetic field of the Universe vacuum. In which, G E M are three orthogonal physical space, and h c G are three unvarying universal fundamental constants. Denoting by λ, υ, c and E_υ the wave-length, oscillation frequency, propagation velocity and the oscillation energy quantum of the propagation G•EM wave in the Universe vacuum, respectively; we can write

$$\upsilon \cdot \lambda = c \tag{1}$$

$$E_{\upsilon} = h\upsilon \tag{2}$$

where h is the Planck constant, c is the light velocity in vacuum. In contrast, denoting by r, ω, c, and E_ω the vortex-radius, whirl frequency (*i.e.*, angular-frequency), peripheral whirl speed and the whirl energy of the primary basic gravitational EM-vortex quantum in the Universe vacuum, respectively, we can write

$$\omega \cdot r = c \tag{3}$$

$$E_{\omega} = \leftarrow_{\omega} \omega \tag{4}$$

Eqs. (3) and (4) are the EM-vortex relation and its whirl energy, respectively, here $\leftarrow = h/2\pi$. (Note: υ and $\omega = 2\pi\upsilon$ refer to different frequencies.)

(B) For the above primary basic energy packet of the dualistic gravitational EM-vortex, the Einstein's mass-energy relation is to hold with a new form as:

$$\leftarrow\omega = m \cdot c^2 \quad (5)$$

where m is the gravitational rest-mass, concentrated into the EM-vortex central point, and $\leftarrow\omega$ is the whirl energy of the EM-vortex quantum. So we call the basic energy packet of the dualistic gravitational EM-vortex as "the primary basic quantum particle of gravitational electro-magnetic vortex (G•EM-vortex)".

(C) The masses of the homogeneous primary basic G • EM-vortices attract one another and the relative gravitational potential energy is $-G \cdot m^2 / 2r$, (where G is the universal gravitational constant); the reduced whirl frequency is ω_R (See Fig.1 for two coupled vortices).

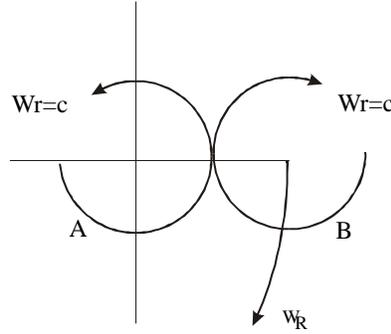


Fig.1 Potential energy of gravitational attraction $G \cdot m^2 / 2r$ are balanced by the whirl energy $\leftarrow\omega_R$ between two similar vortices. Then following the argument for zero total energy first clarified by Edward P. Tryon^{<6>}, the reduced whirl energy $\leftarrow\omega_R$ and the relative gravitational potential energy $-G \cdot m^2/2r$ must satisfy the following relation for a vacuum field fluctuation structure^{<6>}, so net energy of our Universe is zero:

$$\leftarrow\omega_R - G \cdot m^2 / 2r = 0. \quad (6)$$

Because $\omega_R \cdot 2r = c$, and $\omega \cdot r = c$, so

$$\omega_R = \omega/2. \quad (7)$$

And we may have $\leftarrow\omega - G \cdot m^2 / r = 0. \quad (8)$

Eq. (8) is the dualistic G•EM-vortex equation of the basic energy packet. Because $\omega \cdot r = c$, Eq. (8) can be rewritten as:

$$m = (\leftarrow \cdot c/G)^{1/2} \quad (9)$$

and because $\leftarrow\omega = m \cdot c^2$, Eq. (8) can also be rewritten as

$$m \cdot c^2 - G \cdot m^2 / r = 0 \quad (10)$$

i.e., $r = (G/c^2) \cdot m \quad (11)$

or $\omega \cdot c = G \cdot m/r^2 \quad (12)$

i.e., $c^2/r=G \cdot m/r^2 \quad (13)$

So, Eqs.(8), (9), (10), (11), (12), and (13) illustrate that a primary basic quantum particle of energy $m \cdot c^2$ is constructed out of a quantized basic $G \cdot EM$ -vortex $\leftarrow \omega$ (in Universe-vacuum) which will hold itself together by its own gravitational attraction for a very short period of time $2\pi/\omega$ (See Eqs. 21 and 22). The gravitational acceleration needed to hold the $G \cdot EM$ -vortex in a circular orbit of radius r is of the order of G/c^2 . The acceleration available from the gravitation pull of a concentration of whirl energy $\leftarrow \omega$ of mass m is of the order $G \cdot m/r^2$. The two accelerations agree in order of magnitude when the radius r is of the order $r = Gm/c^2$, but the primary particle mass m of the order of $(\leftarrow c/G)^{1/2}$ is described only by the constant quantities h , c , and G , *i.e.*, the quantized $G \cdot EM$ -vortex $\leftarrow \omega$ constructs the primary basic quantum particle mass m without any massive object but the gravitational electromagnetic field. (See Eq.9). In brief, on the collection of the primary quantized electromagnetic vortices, each $\leftarrow \omega$ of which is held together in the way of gravitational attraction $\leftarrow \omega/r$, *i.e.*, $G \cdot m^2/r^2$, (See Eqs.10 and 8), the primary "gravitational electromagnetic vortex ($G \cdot EM$ -vortex) entity" concentrates the primary classical object m into the EM-vortex central point.

Based on the above three fundamental assumptions (A), (B) and (C), and on the three unvarying universal constants h , c , and G mentioned above, for the primary EM-vortex in the Universe vacuum, from the dualistic $G \cdot EM$ -vortex equation (8) of the basic energy packet, with its mass-energy relation (5), and its basic EM-vortex relation (3), we can get a set of the classical Universe equations, formed by the primary particle m just as a primordial cosmos:

$$G \cdot m / c^2 \cdot r = 1 \quad (14)$$

and its intrinsic EM-vortex peripheral whirl relation:

$$\omega \cdot r = c \quad (15)$$

also, we can get a set of the Universal quantum-gravitation equation, formed by the primary basic quantum particle m , namely:

$$G \cdot m^2 / \leftarrow \cdot c = 1 \quad (16)$$

and its quantum relation is easy to deduce from mass-energy relation $\omega = mc^2$:

$$m \cdot c \cdot r = \leftarrow \quad (17)$$

Then, from Eq. (17), we can get the rest-mass m of this primary basic quantum particle

$$m = (\leftarrow \cdot c / G)^{1/2} = 2.18 \times 10^{-5} \text{ g} \quad (18)$$

whose intrinsic energy is

$$m \cdot c^2 = (\leftarrow \cdot c^5 / G)^{1/2} = 1.22 \times 10^{19} \text{ GeV} \quad (19)$$

and from Eqs.(19), (5) and (3), we can get the EM-vortex radius r and the EM-vortex whirl frequency ω of this primary basic quantum particle, respectively,

$$r = (G \cdot \leftarrow / c^3)^{1/2} = 1.62 \times 10^{-33} \text{ cm} \quad (20)$$

$$\omega = (c^5 / G \cdot \leftarrow)^{1/2} = 1.86 \times 10^{43} \text{ sec}^{-1} \quad (21)$$

Here, the EM-vortex radius $r = 10^{-33}$ cm is just equal to the minimum length which is the so called "Planck-length"; so the EM-vortex radius r is the "Planck-radius r_{\leftarrow} " of the primary basic quantum particle's EM-vortex, then the rest-mass of the primary basic quantum particle $m [=(\leftarrow c/G)^{1/2}]$, which is unexplained while using <Gravitation and Cosmology>[3] is the "Planck-mass m_{\leftarrow} ", the intrinsic energy of the primary basic quantum particle mc^2 is the "Planck-energy $m_{\leftarrow}c^2$ ", the EM-vortex whirl frequency ω is related to the "Planck-frequency ω_{\leftarrow} " of the primary basic quantum particle's EM-vortex; and also the "Planck-time t_{\leftarrow} " of the primary basic quantum particle's EM-vortex just as follows:

$$t_{\leftarrow} = 2\pi r_{\leftarrow} / c = 2\pi / \omega_{\leftarrow} = 2\pi(G \cdot \leftarrow / c^5)^{1/2} \quad (22)$$

$$= 3.39 \times 10^{-43} \text{ sec}$$

In short, the primary particle-like concentration of whirl energy with the basic gravitational EM-vortex is just the "Planck-particle" (\leftarrow -particle).

Furthermore, the mass-density and the energy-density of the "Planck-particle" are respectively given as

$$m_{\leftarrow} = m_{\leftarrow} / (4\pi/3) \cdot r_{\leftarrow}^3 = 1.23 \times 10^{93} \text{ gcm}^{-3} \quad (23)$$

$$m_{\leftarrow} \cdot c^2 = m_{\leftarrow} c^2 / (4\pi/3) \cdot r_{\leftarrow}^3 = 1.11 \times 10^{107} \text{ J} \cdot \text{cm}^{-3} \quad (24)$$

and the intrinsic temperature of the "Planck-particle" is

$$T_{\leftarrow} = (m_{\leftarrow} \cdot c^2 / a)^{1/4} = 1.1 \times 10^{32} \text{ }^\circ\text{K} \quad (25)$$

where 'a' is the Black-body constant.

All of these are the fundamental dimensional quantities that appear in gravity, being obtainable by equating the gravitational potential energy to the rest-mass energy. These have recently been recognized as natural units for the whole field of particle physics, leading to the current super-gravity models of super-unification of all fundamental interactions.

In the view of the above results regarding the intrinsic properties of the Planck-particles, such Planck-particles may be taken to constitute just the "Big Bang" sight of the Primordial Cosmoses within the first 10^{-43} sec, with the maximum temperature 10^{32} °K, which were described by S. Weinberg in his book "The First Three Minutes", 1977. Therefore, the "primary particle-like concentration of whirl energy with the Basic G • EM-vortex" is just the "Primordial Cosmos". Namely, "Primordial Cosmos" is just the "Planck quantum-particle of the Primordial Cosmos" itself, and the "Primordial Cosmos" may be called "Planck-Cosmos" (\leftarrow -Cosmos), but in all of them, there are no singularities. Then it may be detailed as follows: from Eqs.(14) and (15), we have a set of classical universe-equation for every single "Primordial Planck-Cosmos", and the sum total of innumerable Primordial Cosmoses constitute a collection of Planck-particles in the Universe-vacuum:

$$G \cdot M_{\leftarrow} / c^2 \cdot R_{\leftarrow} = 1 \quad (26)$$

$$\leftarrow \cdot R_{\leftarrow} = c \quad (27)$$

Also, from Eqs.(16) and (17), we have a set of the universal quantum-gravitation equation for the single "Planck quantum-particle" of every single "Primordial Planck-Cosmos":

$$G \cdot m_{\leftarrow}^2 / \leftarrow \cdot c = 1 \quad (28)$$

and its quantum relation is easy to deduce from mass-energy relation $\leftarrow = m_{\leftarrow} c^2$:

$$m_{\leftarrow} \cdot c \cdot r_{\leftarrow} = \leftarrow \quad (29)$$

Here, M_{\leftarrow} , R_{\leftarrow} and \leftarrow are the whole mass, EM-vortex radius, and angular-frequency of the single "Planck-Cosmos", respectively, and m_{\leftarrow} , r_{\leftarrow} and ω_{\leftarrow} are rest-mass, EM-vortex radius, and angular-frequency of the single "Planck-particle", respectively. And from Eqs. (14), (15), (16) and (17), we can consider that the "Primordial-Cosmos" being just the "Planck-particle" itself, so that $\leftarrow = \omega_{\leftarrow}$, $R_{\leftarrow} = r_{\leftarrow}$ and $M_{\leftarrow} = m_{\leftarrow}$. Then, Eq. (28) can be rewritten as

$$G \cdot m_{\leftarrow}^2 / \leftarrow \cdot c = \leftarrow / \omega_{\leftarrow} \quad (30)$$

(Note: this is the reason, why the ratio $\leftarrow / \omega_{\leftarrow}$ can not in general replace $\omega_{\leftarrow} / \leftarrow$, and should be seen in the deduction of the "present Cosmos quantum-particle" equations.)

And Eq. (28) can also be rewritten as

$$m_{\leftarrow}^3 = \leftarrow^2 / G \cdot c \quad (31)$$

$$i.e., \quad m_{\leftarrow}^3 = \leftarrow^2 / G \cdot R_{\leftarrow} \quad (32)$$

In short, the fact deduced above is that the three constant quantities G , h , c and the gravitational electromagnetic vortex with angular frequency \leftarrow should construct a Cosmos and its quantum-particle. Under the condition that the Primordial Cosmos is its own quantum-particle *i.e.*, $M_{\leftarrow} = m_{\leftarrow}$, $\leftarrow = \omega_{\leftarrow}$, and $R_{\leftarrow} = r_{\leftarrow}$, Eq.(32) is just the same as Eq. (18), and only the constant quantities G , h , c are required to construct the primary Planck-particle mass, *i.e.*, the minimum gravitational EM-vortex with the minimum radius of Planck-length for the Primordial Planck-Cosmos.

In the spirit of the Perfect Cosmological Principle^{<7>}, there should be innumerable Primordial \leftarrow -Cosmoses uniformly distributed in our Universe distinct with all the others to begin with, and it can be calculated that there are about 10^{67} Primordial \leftarrow -Cosmoses as a collection of 10^{67} Planck quantum-particles (\leftarrow -particles) in a volume with the total mass of $\sim 3 \times 10^{62}$ g and the radius of electron's Compton wave-length $/ 2\pi$.

Present Cosmos Universal Quantum-Gravitation Equation and Classical Universe-Equation

As time goes on, the Primordial Planck-Cosmoses annex each another and expand suddenly, so it is called the "sudden expanding", and at the same time, the macro Cosmoses become distinct from their own basic quantum micro particles, such processes may be declared as:

$$m_e [m_{\text{quantum particle}} [m_{\leftarrow} \equiv M_{\leftarrow} [M_{\text{classical Cosmos}} [M_{\text{universe}}$$

where M_{universe} is the rest-mass 3×10^{62} g of our Universe as a whole, m_e is the rest-mass of an electron; and the rest-mass of the Primordial Planck-particle m_{\leftarrow} , *i.e.*, the rest-mass of Primordial Planck-Cosmos M_{\leftarrow} , so, $m_{\leftarrow} = M_{\leftarrow}$ equal to 2.18×10^{-5} g.

And the relation between the micro original quantum particle and the macro Cosmos electro-magnetic vortex can be obtained from Eq. (31) as

$$(m_{\text{quantum}})^3 = \leftarrow^2 / G \cdot c \quad (33)$$

$$i.e., (m_{\text{quantum}})^3 = \leftarrow^2 / G R_{\text{Cosmos}} \quad (34)$$

For the present, our Cosmos whose original basic quantum particle is the pi meson, so it is called the π -cosmos, the relation between the micro quantum particle pion and the macrocosm π -cosmos can be obtained from Eq. (33) as

$$m_{\pi}^3 = \leftarrow^2 / G \cdot c \quad (35)$$

$$i.e., m_{\pi}^3 = \leftarrow^2 / G R_{\pi} \quad (36)$$

Just like all the elementary particles, the quantum particle pion can also be constructed out of an electromagnetic vortex ring, and also can be made to conform the Einstein's mass-energy relation with a new form of quantum-gravitation in $G \cdot EM$ field, so that it is easy to deduce the quantum-gravitation mass-energy relation from the pion's quantum relation $m_{\pi} \cdot c \cdot r_{\pi} = \leftarrow$ as:

$$\leftarrow \omega_{\pi} = m_{\pi} c^2 \quad (37)$$

And then, from Eqs. (35) and (37), we can rewrite Eq. (35) as

$$G \cdot m_{\pi}^2 / \leftarrow \cdot c = \leftarrow / \omega_{\pi} \quad (38)$$

(Note: in order to satisfy the limiting condition of minimum Planck-length, the ratio $\leftarrow / \omega_{\pi}$ in Eq. (38) can not be replaced by $\omega_{\pi} / \leftarrow$).

Here, Eq. (38) and Eq. (37) constitute a set of the universal π -Cosmos quantum-gravitation equation and its quantum relation. And their forms are just the same as that of the Primordial \leftarrow -Cosmos quantum gravitation equation and its quantum relation such as Eqs. (30) and (29), respectively.

As for the π -Cosmos gravitational EM-vortex of radius R_π and total rest-mass M_π , which was annexed and expanded to $(r_\pi/r_{\leftarrow})^3$ times of that of the Primordial \leftarrow -Cosmos, *i.e.*, we have the expansion relation $R_\pi = R_{\leftarrow} (r_\pi/r_{\leftarrow})^3$ and the annexation relation $M_\pi = M_{\leftarrow} (r_\pi/r_{\leftarrow})^3$; then, a set of the π -Cosmos classical Universe-equations can be deduced from that of the Primordial \leftarrow -Cosmos:

$$1 \quad G \bullet M_{\leftarrow} / c^2 \bullet R_{\leftarrow} = G \bullet M_{\leftarrow} (r_\pi / r_{\leftarrow})^3 / c^2 \bullet R_{\leftarrow} (r_\pi / r_{\leftarrow})^3 = G \bullet M_\pi / c^2 R_\pi$$

$$i.e., G \bullet M_\pi / c^2 R_\pi = 1 \quad (39)$$

and using Eqs. (29), (31), (34), and (36), it is easy to deduce the relation $r_\pi / r_{\leftarrow} = (r_{\leftarrow} / r_\pi)^3$; then, together with the expansion relation $R_\pi / R_{\leftarrow} = (r_\pi / r_{\leftarrow})^3$, we can deduce from Eq. (27):

$$c = \leftarrow \bullet (r_{\leftarrow} / r_\pi)^3 \bullet (r_\pi / r_{\leftarrow})^3 \bullet R_{\leftarrow} = \leftarrow (r_\pi / r_{\leftarrow}) (R_\pi / R_{\leftarrow}) \bullet R_{\leftarrow}$$

i.e., the π -Cosmos EM-vortex whirl relation is:

$$\leftarrow \bullet R_\pi = c \quad (40)$$

Here, the form of the π -Cosmos classical Universe-equations, Eqs. (39) and (40), are also the same as that of the Primordial Cosmos such as Eqs.(26) and (27), respectively.

Because $\omega_\pi \bullet r_\pi = c$, Eq.(38) can be rewritten as

$$\leftarrow \bullet \pi \quad G \bullet m_\pi^2 / r_\pi = 0 \quad (41)$$

and together with Eq.(35), it gives

$$m_\pi = (\leftarrow \bullet \pi / G \bullet c)^{1/3}. \quad (42)$$

Because $\leftarrow \bullet R_\pi = C$, Eq.(39) $G \bullet M_\pi / C^2 \bullet R_\pi = 1$ can be rewritten as

$$M_\pi \bullet C^2 \quad G \bullet M_\pi^2 / R_\pi = 0 \quad (43)$$

$$i.e., R_\pi = (G / C^2) \bullet M_\pi$$

or

$$\leftarrow \bullet c = G \bullet M_\pi / R_\pi^2 \quad (44)$$

$$i.e., C^2 / R_\pi = G \bullet M_\pi / R_\pi^2$$

Here, Eq.(41) is the dualistic $G \bullet EM$ -vortex equation of the π -Cosmos. And Eqs. (41), (42), (43) and (44) illustrate that the pion mass m_π and pion radius r_π of the π -Cosmos quantum- particle would be varied with the quantized π -

Cosmos $G \cdot EM$ -vortex $\leftarrow \pi$ (in Universe-vacuum) which will hold itself together by its own gravitational attraction for a very long period of time $2\pi / \omega_\pi$ (See Eqs. 42 and 44). The gravitational acceleration needed to hold the π -Cosmos $G \cdot EM$ -vortex in a circular orbit of radius R_π is of the order of C^2 / R_π . The acceleration available from the gravitational pull of a concentration of π -Cosmos whirl energy $\leftarrow \pi$ of total mass M_π is of the order $G \cdot M_\pi / R_\pi^2$. The two accelerations agree in order of magnitude when the radius R_π is of the order $R_\pi = G \cdot M_\pi / C^2$, but the π -Cosmos basic quantum pion particle mass m_π is described only by the constant quantities h, c, G and the angular-frequency ω_π of π -Cosmos $G \cdot EM$ -vortex of the order of $(\leftarrow \pi / G \cdot C)^{1/3}$, namely, the π -Cosmos $G \cdot EM$ -vortex $\leftarrow \pi$ constructs the π -Cosmos basic quantum particle of pion mass m_π without any other massive object but the gravitational electromagnetic field and the π -Cosmos $G \cdot EM$ -vortex angular-frequency ω_π (See Eq.42). In brief, on the collection of π -Cosmos electromagnetic vortices, each $\leftarrow \pi$ of which is held together in the way of gravitational attraction $G \cdot M_\pi / R_\pi^2$, *i.e.*, $\omega_\pi C$ (See Eq.44), the π -Cosmos "gravitational electromagnetic vortex ($G \cdot EM$ -vortex) entity" concentrates not only the pion quantum particle mass m_π , but also the total mass M_π of the classical π -Cosmos as a whole.

Taking the mass-value of pion, we calculate the radius R_π the total rest-mass M_π , the total energy $M_\pi c^2$, and the mass-density ρ_{M_π} of π -Cosmos:

$$R_\pi = 1.08 \times 10^{27} \text{ cm}$$

$$\text{And } M_\pi = 1.46 \times 10^{55} \text{ g}$$

$$M_\pi C^2 = 1.3 \times 10^{69} \text{ Joule}$$

$$\rho_{M_\pi} = 2.77 \times 10^{-27} \text{ g} \cdot \text{cm}^{-3}$$

It is of interest to note that all these figures not only agree with that of Einstein's steady-state Universe, but also fall in the range of values estimated by J.A.Wheeler in his classical theory of the "geons". So we consider that the present Cosmos is the π -Cosmos and may be called "the present π -Cosmos". And it can be calculated that there are about 10^7 π -Cosmos (their total mass $\sim 3 \times 10^{62}$ g) still equals to that of 10^{67} \leftarrow -Cosmos) in our Universe (not only). So at present, the cosmoses are still annexing and expanding.

As for, the annexing cosmoses various stages of our Universe the universal form of the classical Universe-cosmos equation is:

$$G \cdot M / C^2 \cdot R = 1 \quad (45)$$

and its EM-vortex whirl relation is:

$$\omega \cdot R = C \quad (46)$$

the Universal quantum-gravitation equation is

$$G \cdot M^2 / \leftarrow \cdot c = \leftarrow / \omega \quad (47)$$

$$i.e., m = (\leftarrow^2 / G \cdot c)^{1/3} \quad (48)$$

and its quantum relation is: $m \cdot c \cdot r = \leftarrow \quad (49)$

where M, R and \leftarrow are the total rest-mass, the radius and the angular-frequency of the whole Cosmos gravitational EM-vortex, respectively, and where m, r and ω are the rest-mass, radius and the angular - frequency of the annexing Cosmos basic quantum - particle EM-vortex, respectively.

Large Number Hypothesis and the fine-structure constant

On the basic of the above deduction and quantitative argument, we obtain exactly the present π -Cosmos quantum-gravitation equation (38), *i.e.*, Eq.(35) such as $m_\pi = (\leftarrow^2 / G \cdot c)^{1/3}$. And this quantum-gravitation equation (35) no longer appears as a hypothesis, so it is not only different from Dirac' s Hypothesis, *i.e.*, $m_\pi \sim (\leftarrow^2 H / G \cdot C)^{1/3}$, but also shows directly the single "Large Number" to hold covariant for the coupling relation between the micro quantum-gravitation of the pion quantum particle and the cosmos scale EM-vortex of classical π -Cosmos. Also, it appears to offer a quantitative argument for the origin of our "Large Number" on the basic of the gravitational electromagnetic vortex structure of Universe vacuum and the vacuum fluctuation. Then, our single "Large Number" can be calculated with the mass-value of pion from Eq.(35), *i.e.*, Eq.(38):

$$\begin{aligned} \leftarrow \cdot c / G \cdot m_\pi^2 &= \omega_\pi / \pi \quad (50) \\ &= 7.65 \times 10^{39} \end{aligned}$$

Correspondingly, one of Dirac' s "Large Number" is calculated with that of the mass-value of proton:

$$\leftarrow \cdot c / G \cdot m_p^2 = 1.7 \times 10^{38} \quad (51)$$

In addition, the ratio between Eq. (39) and Eq. (38) appears to offer the square of our "Large Number":

$$M_\pi / m_\pi = (7.65 \times 10^{39})^2 \quad (52)$$

Correspondingly, another one of Dirac' s "Large Number" is calculated with the ratio between the present static-state Cosmos total mass M and the proton mass m_p :

$$M / m_p = (3.4 \times 10^{39})^2 \quad (53)$$

In short, our "Large Number" is based on \leftarrow and m_π , whereas Dirac' s "Large Number" are based on H and m_p . In this way, we can construct another two relations of "Large Number" just as Dirac did, and also hold intact the value of our single "Large Number":

Dirac' s Large Numbers	Our single Large Number of π -Cosmos Present
(a)* $m_e \cdot c^3 / e^2 H = 7 \times 10^{39}$	$2m_e \cdot c^3 / e^2 \pi = 7.65 \times 10^{39}$
(b) $e^2 / G \cdot m_p \cdot m_e = 2 \times 10^{39}$	$e^2 / 2 G \cdot m_\pi \cdot m_e = 7.65 \times 10^{39}$

* Dirac' s another manifestation of this "Large Number" being

$$c \cdot t_{\text{Universe}} / (e^2 / m_e \cdot c^2) = 4 \times 10^{40}.$$

Then, we can say that our single "Large Number" no longer appears as an unexplainable and inexplicable hypothesis but also shows the coupling relation of the classical macro cosmos and its own micro quantum-particle, and determined by the Universal quantum-gravitation equation, especially it is related to the Universe evolution stage. As for the primordial Cosmos, *i.e.*, every non-singularity Primordial Planck-Cosmos is just its own Planck quantum-particle, so every primary "Large Number" of the self-coupling Planck-Cosmos is merely 1; and then, about 10^{67} Planck-Cosmos of our Primordial Universe annex and expand till now. At the present, our π -Cosmos "Large Number" is increased to $(7.65 \times 10^{39})^2$, (see Eq.52), finally, the "Large Number" of non-divergence chaos-Universe will be increased to $M_e/m_e = (5.69 \times 10^{44})^2$

Using our Single Large Number (a) in the above table, and from Eq. (50), we can get

$$2 m_e \cdot c^3 / e^2 \cdot \pi = \omega_\pi / \pi \quad (54)$$

$$1 / \omega_\pi = e^2 / 2m_e \cdot c^3 \quad (55)$$

It means that the unit of time for pion is equal to $e^2 / 2m_e \cdot c^3$, *i.e.*, it is half of the atomic time unit $e^2 / m_e \cdot c^3$, which has often been remarked. And Eq. (53) shows us that the physical meaning of our "Large Number" is the ratio of the π -Cosmos time unit $1 / \pi$ to the pion time unit $1 / \omega_\pi$; or the ratio of the π -Cosmos scale R_π to the pion scale r_π . And its micro-physical presentation is the "fine-structure constant", so we can get the mass relation of the "fine-structure constant" from it:

$$e^2 / \leftarrow \cdot c = 2 m_e / m_\pi \quad (56)$$

Similarly, the mass relation of the "fine-structure constant" can also be obtained from our Singer Large Number (b) in the above table, and Eq.(50) yields:

$$e^2 / 2G \cdot m_\pi \cdot m_e = \leftarrow \cdot c / G \cdot m_\pi^2 \quad (57)$$

namely,
$$e^2 / \leftarrow \cdot c = 2 m_e / m_\pi \quad (58)$$

where the physical meaning of our "Large Number" is the ratio of the electric force (between two atomic electrons) to the gravitational force (between pion meson and atomic electron). And also, it is restricted by the pion quantum-particle Universe-equation (38) and the "constant of fine-structure" relation (58).

However, these results show us that the set of π -Cosmos $G \bullet EM$ -vortex Universe-equations (39) and (40), together with the set of pion quantum-relation (37) and π -Cosmos quantum-gravitation equation (38), restrict the "constant" of fine-structure relation, and also the coupling relation of π -Cosmos and its own pion quantum-particle on space-time scale of "classical macro π -Cosmos" and "micro pion-quantum". All of these are determined by the Universe evolution stage which is described by our classical Universe-equation and Universal Quantum-gravitation equation. And it is represented by our single "Large Number" (the function of the Universe evolution stage).

Clearly, it is shown that there is also the Mach's Principle which reveals why equations like Eqs. (38) and (39) are determined not solely by considerations of microphysics, but in nature also by the influence of the whole Universe's Mass and the whole Universe's gravitational electromagnetic vortex ($G \bullet EM$ -vortex) whirl frequency.

Furthermore, it is shown here that Dirac's Hypothesis of $m\pi \sim (\leftarrow 2H / GC)^{1/3}$ is correct and similar to our present π -Cosmos quantum-gravitation equation of $m_\pi = (\leftarrow^2 \pi / GC)^{1/3}$,

Namely,

$$\frac{G \cdot m_\pi^2}{\eta \cdot c} = \frac{\Omega_\pi}{\omega_\pi} \quad (59)$$

The "Great Thundering" of Chaos-Universe
and Chaos Fusion

About 1.35×10^{67} Planck Cosmosees annex each other and expand. Finally, for the electron Universe, which is called the chaos Universe, its "Large Number" increases to $M_e / m_e = (5.69 \times 10^{44})^2$. This means that there are $(5.69 \times 10^{44})^2$ electrons in the electron chaos Universe, where m_e is the mass of electron, M_e is the total mass of the whole electron-Universe, and M_e is about 3×10^{62} g.

Then, the classical Universe-equation of chaos electron-Universe, just like Eq. (39) becomes:

$$GM_e / c^2 R_e = 1 \quad (60)$$

and its EM-vortex whirl relation is:

$$\omega_e \bullet R_e = c \quad (61)$$

ω_e is the whirl frequency of the EM-vortex of chaos electron-Universe as a whole and R_e is its radius.

Moreover, the Universal quantum-gravitation equation of chaos electron-Universe, similar to Eq. (38), is:

$$Gm_e^2 / \leftarrow c = \omega_e / \omega_e \quad (62)$$

The quantum relation of electron's EM-vortex is

$$m_e \bullet c \bullet r_e = \leftarrow \quad (63)$$

where ω_e is the whirl frequency of the electron's EM-vortex and r_e is its radius. From Eqs. (60), (62) and (63), we have the chaos electron-Universe mass coupling constant ($\leftarrow c / G$) and the relation is:

$$\pm (M_e \bullet m_e^3)^{1/2} = \leftarrow c / G \quad (64)$$

From this relation and under the conditions of Eqs. (45), (49) and Eq. (28), it follows that

$$r_e^3 / R_e = r_{\leftarrow}^2 \quad (65)$$

namely

$$r_e^3 / R_e^3 = r_{\leftarrow}^2 / R_e^2 \quad (66)$$

where r_{\leftarrow} is the radius of Plank-Cosmos, it is called the ‘‘Servo mechanism’’. Eq. (65) and Eq. (66) are the same relation as the chaos Universe’s Servo-mechanism, which states that the chaos electron-Universe returns to Primordial Universe of 1.5×10^{67} Plank particle-Cosmoses. For each of them, the following relations hold:

$$\begin{aligned} M_{\leftarrow} &= m_{\leftarrow} \\ M_{\leftarrow} c^2 &= m_{\leftarrow} c^2 = \leftarrow \omega_{\leftarrow} = \leftarrow \leftarrow \quad (67) \\ \omega_{\leftarrow} \cdot r_{\leftarrow} &= c \\ \leftarrow \cdot R_{\leftarrow} &= c \\ m_{\leftarrow} \cdot c \cdot r_{\leftarrow} &= \leftarrow \end{aligned}$$

and the classical Primordial \leftarrow -cosmos equation $GM_{\leftarrow} / c^2 R_{\leftarrow} = 1$, and Primordial \leftarrow -cosmos quantum-gravitation equation $Gm_{\leftarrow}^2 / \leftarrow c = \leftarrow / \omega_{\leftarrow}$, *i.e.*, $Gm_{\leftarrow}^2 / \leftarrow c = 1$. They are just the same as in Eq. (26), (30), and (28). Then, 1.35×10^{67} m_{\leftarrow} concentrate on a small volume with the radius scale of r_e . Thus, it had to begin with annexing and expanding and to get a ‘‘sudden expanding’’, which is so called the ‘‘Big Bang’’. However, all of the 1.35×10^{67} Planck-particle cosmoses of our Primordial Universe annex and expand once again.

From its Classical Universe-Equation (Eq.45) and Universal Quantum-Gravitation Equation (Eq.47), we can get for the annexing Cosmos the Universal Mass (of Cosmos M and Basic Particle m) and the Coupling Constant $\leftarrow c / G$. And the Universal Mass-Coupling Constant Relation is:

$$\leftarrow c / G = \pm (M \cdot m^3)^{1/2} \quad (68)$$

For \leftarrow -Cosmos, Eq. (68) would be written as:

$$\leftarrow c / G = \pm (M_{\leftarrow} \cdot m_{\leftarrow}^3)^{1/2} \quad (69)$$

because $M_{\leftarrow} = m_{\leftarrow} \quad (70)$

So $Gm_{\leftarrow}^2 / \leftarrow c = 1 \quad (71)$

i.e., $(\leftarrow c / G)^{1/2} = m_{\leftarrow} \quad (72)$

Chaos Universe, full of $(5.69 \times 10^{44})^2$ Fermion electron, annihilate with ‘‘Great Thundering’’ to Universal Boson electron (e_0) Universe, and the r ring with vortex radius R_{e_0} , without ‘‘heat death’’:

$$e^+ + e^- \rightarrow e_0 + r \quad (73)$$

that e_0 Universe full of original Boson electron e_0 the original e_0 Universe Mass Coupling Relation is:

$$\leftarrow c / G = \pm (M_{e_0} \cdot m_{e_0}^3)^{1/2} \quad (74)$$

Classical Universe Equation of original e_0 Universe is:

$$G M_{e_0} / C^2 R_{e_0} = 1 \quad (75)$$

Universal Quantum-Gravitation Equation of original e_0 Universe is:

$$m_{e_0}^3 = \leftarrow^2 / G R_{e_0} \quad (76)$$

$$m_{e_0}^3 = \leftarrow^2 / G C \quad (77)$$

and original m_{e_0} -Quantum relation is

$$m_{e_0} \cdot c \cdot r_{e_0} = \leftarrow \quad (78)$$

From above Eqs. (76), (77), and (78), there being the servo-mechanism of original e_0 Universe:

$$r_{\leftarrow}^2 \cdot R_{e_0} = r_{e_0}^3 \quad (79)$$

$$\text{where } r_{\leftarrow}^2 = Gh / c^3 \quad (80)$$

which come from the Universal Quantum-Gravitation Equation of \leftarrow -Cosmos:

$$m_{\leftarrow}^3 = \leftarrow^2 / GR_{\leftarrow} \quad (81)$$

$$= \leftarrow^2 / Gr_{\leftarrow}$$

$$i.e., \quad Gm_{\leftarrow}^2 / \leftarrow c = 1 \quad (82)$$

and the Servo-mechanism can be written as:

$$r_{e_0}^3 / R_{e_0}^3 = r_{\leftarrow}^2 / R_{e_0}^2 \quad (83)$$

this Servo-mechanism supports the fusion process of e_0 Universe, to produce Planck-particle cosmoses of primordial universe with radius r_{e_0} , in which, there are about 1.35×10^{67} \leftarrow -Cosmoses gathering together.

Servo-System of original Quantum particle coupling relation

The primordial first period coupling relation between boson status original quantum particle of Planck (\leftarrow), Electron (e_0), Primordial Pion π_{01} Primordial Neutron n_{01} , and the second period Hyperon (n_{02}) denoting by their radius r_h , r_{e_0} , $r_{\pi_{01}}$, $r_{n_{01}}$ and $r_{n_{02}}$ the first period servo-system of coupling relation can be shown as:

$$r_{\leftarrow}^2 \cdot r_{e_0} = r_{\pi_{01}}^3 \quad (84)$$

$$r_{\leftarrow}^2 \cdot r_{\pi_{01}} = r_{n_{01}}^3 \quad (85)$$

$$r_{\leftarrow}^2 \cdot r_{n_{01}} = r_{n_{02}}^3 \quad (86)$$

$$\text{and from Eqs. (84) and (85) } r_{n_{01}}^3 \cdot r_{e_0} = r_{\pi_{01}}^4 \quad (87)$$

$$\text{from Eq. (84), (85), and (86) } r_{\leftarrow}^{26} \cdot r_{e_0} = r_{n_{01}}^{27} \quad (88)$$

Here, for Planck-particle is the Primordial Hyperon n_{01} , its radius r_{\leftarrow} can be rewritten as $r_{n_{01}}$, so there is

$$r_{n_{01}}^{26} \cdot r_{e_0} = r_{n_{02}}^{27} \quad (89)$$

where $r_{n_{02}}$ is the radius of the second period Hyperon n_{02} . Then, Eq. (89) can be rewritten as:

$$r_{nN}^{26} \cdot r_{e_0} = r_{n_{N+1}}^{27} \quad (90)$$

where $N=01, 02, 03, \dots$ are the numerical order of servo-system period when $N=049$. *i.e.*, the 49th period of annex-cosmos, there are:

$$\text{Classical Universe-equation: } G \cdot M_{\pi_{049}} / C^2 R_{\pi_{049}} = 1 \quad (91)$$

$$\text{and } \pi_{049} \cdot R_{\pi_{049}} = C \quad (92)$$

$$\text{Universal quantum-gravitation equation: } G \cdot m_{\pi_{049}}^2 / \leftarrow c = \pi_{049} / \omega_{\pi_{049}} \quad (93)$$

$$i.e., m_{\pi_{049}}^2 = \leftarrow^2 / G R_{\pi_{04}} \quad (94)$$

$$m_{\pi_{049}} \cdot C \cdot r_{\pi_{049}} = \leftarrow \quad (95)$$

According to Eqs. (80), (90) from $m_{01} = r_{\leftarrow}$ to $r_{n_{049}}$.

$$\text{we get: } m_{n_{049}} \cdot C^2 = 2319.03 \text{ MeV}$$

$$m_{\pi_{049}} \cdot C^2 = 140.096 \text{ MeV}$$

$$m_{n_{049}} \cdot C^2 = 909.941 \text{ MeV}$$

$$\text{then } \pi_{049} \rightarrow \pi + \nu_{\mu} \quad (96)$$

$$n_{049} + \pi^0 - \mu_0 \Leftrightarrow n \quad (97)$$

$$n \rightarrow p + e + \nu_{\mu} \quad (98)$$

where ν_μ is neutrino of muon. π is pion meson, n is neutron, π_{049} is the 49th period original pion meson π_0 , n_{04} is the 49th period original neutron i.e., n_0 p is the proton, e is the electron and from

$$m_e \cdot c^2 = 0.511 \text{ MeV}$$

$$m_{\nu_\mu} \cdot c^2 = 0.528 \text{ MeV (include } \nu_\mu \text{ and binding energy)}$$

$$m_{\mu_0} \cdot c^2 = 105.626 \text{ MeV}$$

$$m_{\pi_0} \cdot c^2 = 134.965 \text{ MeV}$$

we get the result: $m_\pi \cdot c^2 = 139.568 \text{ MeV}$

$$m_n \cdot c^2 = 939.280 \text{ MeV}$$

and $m_p \cdot c^2 = 938.241 \text{ MeV}$, they are all consistent of the result of experiments. So our cosmos at present is the 49th period π -cosmos.

Interaction Coupling constants

It is easy to deduce that all kinds of interaction coupling constants is governed by the cosmos Quantum-Gravitation constant:

$$[G_{\leftarrow}^2 / C^2] \cdot [R_{\pi N} / r_{e_0}] \quad (99)$$

from $r_{\leftarrow}^2 \cdot r_{e_0} = r_{\pi 01}^3$, $r_{\leftarrow}^2 \cdot R_{e_0} = r_{\pi 01}^3$, and $m_{\pi_01}^3 = \leftarrow^2 / GR_{\pi 01}$,

we can get: $R_{\pi 01} = r_{e_0} \quad (100)$

So as for the first period, its Quantum-Gravitation constant is:

$$[G_{\leftarrow}^2 / C^2] \quad (101)$$

As for the present π -cosmos, its Quantum-Gravitation constant is:

$$[G_{\leftarrow}^2 / C^2] \cdot (R_{\pi_0} / r_{e_0}) \quad (102)$$

and all kinds of interaction coupling constants are:

$$[G_{\leftarrow}^2 / C^2] (R_{\pi} / r_{e_0}) = [g_o^2(\text{strong})]_{\pi\text{-cosmos}} \cdot r_{n_0}^2 \quad (103)$$

$$= [e_o^2(\text{EM})]_{\pi\text{-cosmos}} \cdot r_{\pi_0}^2 \quad (104)$$

$$= 2[GM_{\pi}^2]_{\pi\text{-cosmos}} \cdot (R_{\pi_0} / r_{e_0}) \cdot r_{\pi_0}^2 \quad (105)$$

where the strong interaction coupling constant is:

$$[g_o^2(\text{strong})]_{\pi\text{-cosmos}} = 2\leftarrow c \cdot (r_{n_0} / r_{\pi_0}) = 2\leftarrow c (m_{\pi_0} / m_{n_0}) \quad (106)$$

$$[e_o^2(\text{EM})]_{\pi\text{-cosmos}} = 2\leftarrow c (r_{\pi_0} / r_{e_0}) = 2\leftarrow c (m_{e_0} / m_{\pi_0}) \quad (107)$$

$[GM_{\pi}^2]_{\pi\text{-cosmos}}$ is the gravity pion with the physical dimension of $[\text{erg} \cdot \text{cm}]$ which can be deduced from

$$m_{\pi} C \cdot r_{\pi} = \leftarrow$$

$$m_{\pi}^2 C^2 r_{\pi}^2 = \leftarrow^2$$

$$[GM_{\pi}^2] = (G_{\leftarrow}^2 / C^2 r_{\pi}^2) \quad (108)$$

Since $r_{\leftarrow}^2 = G_{\leftarrow} / C^3$, the above Eqs. (103) and (104), and Eqs. (106) and (107) consist of Servo-System relations:

$$r_{\leftarrow}^2 \cdot r_{\pi_0} = r_{n_0}^3 \quad (109)$$

$$r_{\leftarrow}^2 \cdot r_{e_0} = r_{\pi_0}^3 \quad (110)$$

and servo-mechanism relation $r_{\leftarrow}^2 R_{\pi_0} = r_{\pi_0}^3 \quad (111)$

Eq. (105)/Eq. (104) gives

$$\frac{2[G m_{\pi}^2]_{\pi\text{-cosmos}} \cdot (R_{\pi o} / r_{e o})}{[e_o^2(\text{EM})]_{\pi\text{-cosmos}}}$$

i.e., $2[G m_{\pi}^2]_{\pi\text{-cosmos}} / [e_o^2(\text{EM})]_{\pi\text{-cosmos}} = r_{e o} / R_{\pi o} = 1 / (2.8 \times 10^{37})$ (112)

Eq. (105)/Eq. (103) gives:

$$\frac{2[G m_{\pi}^2]_{\pi\text{-cosmos}} \cdot (R \pi_o / r_{e_o})}{[g_o^2(\text{strong})]_{\pi\text{-cosmos}}} \cdot \frac{2[G m_{\pi}^2]_{\pi\text{-cosmos}} \cdot (R \pi_o / r_{e_o}) / [e_o^2(\text{EM})]_{\pi\text{-cosmos}}}{[g_o^2(\text{strong})] / [e_o^2(\text{EM})]_{\pi\text{-cosmos}}} \cdot \frac{R \pi_o / r_{e_o}}{(r \pi_o / r n_o)^2} \cdot \frac{R \pi_o / r_{e_o}}{(m n_o / m \pi_o)^2} \cdot \frac{1}{1.2 \times 10^{39}}$$

(113)

All of the above results in Eqs. (112) and (112), are consistent with the physical experiments.

From Eq. (107), we can get the fine-structure constant as:

$$[e_o^2(\text{EM})]_{\pi\text{-cosmos}} / \leftarrow c = 2 r_{\pi o} / r_{e o} = 2 m_{e o} / m_{\pi o} = 1/137.003$$
 (114)

The result of Eq. (113) is also consist with the fine-structure experiment value, *i.e.*:

$$e^2 / \leftarrow c = 1 / 137.036$$
 (115)

Summary

Based on h, c, G, which are three unvarying Universal fundamental constants, we propose the dualistic gravitational electro-magnetic vortex model and the vacuum's fluctuation mechanism. Considering that A.Einstein's mass-energy relation may be shown in gravitational electro-magnetic form, we tried to unite the gravitational field and the electro-magnetic field into a single structure and relying on which to solve the puzzle of Dirac's Large-Number Hypothesis. Especially, we find that primordial Universe comes from chaos Universe's "Great Thundering", not from a singularity, and there are about 10^{67} Planck quantum-particle Cosmoses gathering in a volume of the electron's scale with the total mass of $\sim 10^{67}$, and begin with sudden annexing and expanding, which we call "Sudden Annexing". We also find that our Universe should be finally going to the non-divergent chaos of electron-Universe. Based on the annihilation of electrons, the "Great Thundering" gives a bright ring of high energy r light, according to the servo-mechanism, and the chaos Universe returns to the Primordial Planck Universe with fusion process, as for the present π -cosmos. We find that all kinds of interaction coupling constants are governed by the "Cosmos Quantum-Gravitation Constant", and all of these calculation results consist with physical experiments.

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